1. *"Hello World" in CVX.*
Consider the following complex least ℓ_∞-norm problem (in our mid-term exam):

$$\begin{array}{ll} \text{minimize} & \|x\|_\infty \\ \text{subject to} & Ax = b, \end{array}$$

with *complex* variables $x \in \mathbb{C}^n$ and *complex* parameters $A \in \mathbb{C}^{m \times n}$ and $b \in \mathbb{C}^m$. Note that

$$\|x\|_\infty = \max_{i=1,\ldots,n} |x_i|.$$

Generate the problem data as follows:

- $m = 30$ and $n = 100$
- A and b can be randomly generated according to Gaussian distribution. The Matlab command will be

```
A = randn(m,n) + i*randn(m,n); b = randn(m,1) + i*randn(m,1);
```

Note that Matlab defines variable `i` as `i` $= \sqrt{-1}$ by default.

(a) Use CVX to solve the problem with complex variables directly.
Hint: You can use `norm(x, inf)` with complex arguments directly in CVX. CVX recognizes `norm(x, inf)` as a valid convex function. You need to declare variables to be `complex` in the `variable` statement.

(b) We can convert the problem into a SOCP with real variables as follows (please refer to the solution to the mid-term exam for details on how it is converted). First, define the optimization variable as

$$z = \begin{bmatrix} \mathbf{Re}(x) \\ \mathbf{Im}(x) \end{bmatrix}.$$

Then define

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} \mathbf{Re}(A) & -\mathbf{Im}(A) \\ \mathbf{Im}(A) & \mathbf{Re}(A) \end{bmatrix} \\ \tilde{b} &= \begin{bmatrix} \mathbf{Re}(b) \\ \mathbf{Im}(b) \end{bmatrix} \end{aligned}$$

Finally, define $C_i \in \mathbb{R}^{2\times(2n)}$ as a matrix with all zero elements except $(C_i)_{1,i} = 1$ and $(C_i)_{2,n+i} = 1$, such that

$$\begin{bmatrix} \mathbf{Re}(x_i) \\ \mathbf{Im}(x_i) \end{bmatrix} = C_i \begin{bmatrix} \mathbf{Re}(x) \\ \mathbf{Im}(x) \end{bmatrix}.$$

The equivalent SOCP will be

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & \|C_i z\|_2 \le t, \ i = 1, \ldots, n \\ & \tilde{A}z = \tilde{b}, \end{array}$$

with optimization variables $t \in \mathbb{R}$ and $z \in \mathbb{R}^{2n}$.
Solve the above SOCP using CVX.

(c) Verify the optimal solutions and optimal values obtained in two approaches are the same.

(d) Please print out your codes and submit them with your solution.