Convex Optimization

Lecture 10 - Disciplined Convex Programming and CVX

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The Big Picture of Where We Are

what we have learned:

- basic concepts of convex sets, convex functions, convex optimization problems
- operations that preserve convexity of sets and functions
- equivalent problem formulations
- special classes of convex programs: LP, SOCP, GP, SDP
- duality, KKT conditions

what we have not learned:

- how to solve convex optimization problems numerically

Disciplined Convex Programming

disciplined convex programming (DCP):

- a library of functions and sets (or atoms) with known convexity, monotonicity, etc.
- a ruleset of how to use or combine the atoms

what DCP cannot do:

- check whether the problem is convex or not
- solve arbitrary convex programs

CVX:

- a DCP software (in Matlab and Python)
- documentation and examples online

Atoms - Sets

some constraints expressed as convex sets:

- `X == semidefinite(n)` means $X \in \mathbb{S}_+^n$
- `[X z ; z' t] == semidefinite(n)` means $\begin{bmatrix} X & z \\ z^T & t \end{bmatrix} \geq 0$
- `x == nonnegative(n)` means $x \in \mathbb{R}^n,\ x \geq 0$

Atoms - Convex Functions

some functions recognized by CVX:

function	meaning	attributes
`norm(x,p)`	$\|x\|_p$	convex
`square(x)`	x^2	convex
`square_pos(x)`	x^2 $(x \geq 0)$	convex, nondecreasing
`sum_largest(x,k)`	$x_{[1]} + \cdots + x_{[k]}$	convex, nondecreasing
`sqrt(x)`	$\sqrt{x}$ $(x \geq 0)$	convex, nondecreasing
`inv_pos(x)`	$1/x$ $(x > 0)$	convex, nonincreasing
`max(x)`	$\max\{x_1, \ldots, x_n\}$	convex, nondecreasing
`quad_over_lin(x,y)`	x^2/y $(y > 0)$	convex, nonincr in y
`lambda_max(X)`	$\lambda_{\max}(X)$ $(X = X^T)$	convex

the library of atoms can be extended

DCP Ruleset - Composition

combine atoms using valid composition rules:

- a convex function of an affine function is convex
- the negative of a convex function is concave
- a convex, nondecreasing function of a convex function is convex
- a concave, nondecreasing function of a concave function is concave
- .....

DCP Ruleset - Limitations

the ruleset includes sufficient conditions

some functions are convex, but not recognized by CVX as convex

example: `sqrt( sum( square( x ) ) )`

- a concave, nondecreasing function of a convex function
- workaround: use `norm( x )`

example: `square( 1 + square( x ) )`

- a convex function of a convex function
- workaround: use `square_pos( 1 + square( x ) )`